Instructions: Try these problems from the textbook as practice (answers below).

Problems

1. Find the domain of $\mathbf{r}(t) = \left\langle \ln(t+1), \dfrac{t}{\sqrt{9-t^2}}, 2^t \right\rangle$.

3. Compute $\lim\limits_{t\to 0} \left(e^{-3t}\mathbf{i} + \dfrac{t^2}{\sin^2(t)}\mathbf{j} + \cos(2t)\mathbf{k} \right)$.

5. Compute $\lim\limits_{t\to 0} \left(\dfrac{1+t^2}{t-1}\mathbf{i} + \arctan(t)\mathbf{j} + \dfrac{1-e^{-2t}}{t}\mathbf{k} \right)$.

7-16. Sketch the curve with the given vector equation. Indicate with an arrow the direction in whith t increases.

   7. $\mathbf{r}(t) = \langle -\cos t, t \rangle$
   9. $\mathbf{r}(t) = \langle 3\sin t, 2\cos t \rangle$
   11. $\mathbf{r}(t) = \langle t, 2-t, 2t \rangle$
   13. $\mathbf{r}(t) = \langle 3, t, 2-t^2 \rangle$
   15. $\mathbf{r}(t) = \langle t^2, t^4, t^6 \rangle$

17. Draw the projection of the curve $\mathbf{r}(t) = \langle t^2, t^3, t^{-3} \rangle$ onto the yz-plane.

19. Draw the projections of the curve $\mathbf{r}(t) = \langle t, \sin t, \cos t \rangle$ onto the three coordinate planes. Use these projections to help sketch the curve.

21-24. Find the vector equation and parametric equations for the line segment that joins P and Q.

   21. $P(-2, 1, 0)$, $Q(5, 2, -3)$
   23. $P(3.5, -1.4, 2.1)$, $Q(1.8, 0.3, 2.1)$

31-34. Find an equation of the plane that contains the given curve.

   31. $\mathbf{r}(t) = \langle t, 4, t^2 \rangle$
   33. $\mathbf{r}(t) = \langle \sin t, \cos t, -\cos t \rangle$

35. Show that the curve with parametric equations $x = t\cos t, y = t\sin t, z = t$ lies on the cone $z^2 = x^2 + y^2$. Use this fact to help sketch the curve.

39. At what points does the curve $\mathbf{r}(t) = t\mathbf{i} + (2t - t^2)\mathbf{k}$ intersect the paraboloid $z = x^2 + y^2$?

49. Show that the curve with parametric equations $x = t^2, y = 1 - 3t, z = 1 + t^3$ passes through the points $(1, 4, 0)$ and $(9, -8, 28)$ but not $(4, 7, -6)$.

50-54. Find a vector function that represents the curve of intersection of the two surfaces.

   51. The cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$.
   53. The hyperbolic paraboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.

Answers

1. $(-1, 3)$
3. $\mathbf{i} + \mathbf{j} + \mathbf{k}$
5. $\left\langle -1, \frac{\pi}{2}, 0 \right\rangle$
21. $\mathbf{r}(t) = \langle -2 + 7t, 1 + t, -3t \rangle$ on $0 \leq t \leq 1$
23. $\mathbf{r}(t) = \langle 3.5 - 1.7t, -1.4 + 1.7t, 2.1 \rangle$ on $t \in [0, 1]$
31. $y = 4$
33. $z = -y$
39. $(0, 0, 0)$ and $(1, 0, 1)$
49. $t_1 = -1$; $t_2 = 3$; $t_3 = -2$
51. $\mathbf{r}(t) = \left\langle t, \frac{1}{2}(t^2 - 1), \frac{1}{2}(t^2 + 1) \right\rangle$
53. $\langle \cos(t), \sin(t), \cos(2t) \rangle$ on $0 \leq t \leq 2\pi$